**Question 1: Counting to a Billion**

How long would it take you to count to one billion, reciting the numbers one after another? Write a guess.

Explain your guess. What information did you use to decide how long it would take to count to one billion?

Now, let's use math and logic to come up with a thoughtful answer. First, think about how long it takes to say **each** number. How long does it take to say each number from one to one billion?

Does it take the same amount of time to say each number? If you say yes, explain why. If you say no, explain why not.

Now, how many numbers are there between one and one billion? Using the amount of time it takes to say each number (from above) determine how long it takes to say all of them in order.

Now, take a look at your units. Have you used seconds, minutes, hours, or something different? Does your answer make sense to other people? If you said your answer to someone else, would they understand what the number means? Take your number and convert it to a more manageable answer so that others understand. (Example: If I tell you I will see you in 86,400 seconds, you won't understand what I mean. If I tell you I'll see you *tomorrow* (which is 86,400 seconds away) then you *would* understand.)

**Question 2: Simplifying Expressions**

Simplify the following:

$$3 + 2 + 3 + 2 + 3 + 2 + 3 + 2 + 3 + 2 + 3 + 2 + 3 + 2 + 3 + 2 + 3 + 2$$

What is your answer?

What did you to do solve?

Notice that there are a few ways to solve this. Explain a **different** way to solve than the way that you solved this above.

Now, simplify the following:

$$x + 2 + x + 2 + x + 2 + x + 2 + x + 2 + x + 2 + x + 2 + x + 2 + x + 2$$

What is your answer?

What did you do to solve?

Again, there are a few ways to solve this. Explain a **different** way to solve than the way that you solved this above.

**Question 3: Opposites**

What is the value of 3 + (-3)?

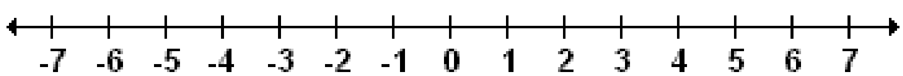
What is the value of (-10.4) + 10.4?

These pairs of numbers are called *opposites*. What is the sum of a number and its opposite?

Does every number have an opposite? If you said yes, explain why. If you said no, explain why not.

Write the opposite of the following numbers:
a) -2.341 b) 1/3 c) 7 d) x challenge e) x + 2 f) x – 2

Now, plot each number and its opposite (a-d from above) on the number line below.




Number Line

What do you think is the definition of an *opposite*? Use what you have just done to come up with a thoughtful definition that is **in your own words**.

Now, what do you think the property (rule) is for adding a number to its opposite?

**Question 4: Distributive Property of Addition**

Your class sponsors a benefit concert and prices the tickets at $8 each. Deonte sells 12 tickets, Andy 16, Morgan 17, and Pedro 13. Compute the total revenue brought in by these four persons.

Notice there are two ways to do this calculation. Show both of them below.

| Method 1 | Method 2 |
|---|---|
| | |

A familiar feature of arithmetic is that *multiplication distributes over addition*. This is called the *distributive property*. Written in algebraic code, this property looks like

$$a(b + c) = ab + ac$$

Give an example of the distributive property using numbers.

Now, think of a **real-life situation** where you would use the distributive property. Explain that situation below using words and numbers. (No, you may not use the first problem on this page as your example).

What is $x(2 + 5)$? Show two ways to solve.

**Question 5: Reciprocals**

The division problem $12 \div \frac{3}{4}$ is equivalent to the multiplication problem $12 \cdot \frac{4}{3}$. Explain this. You can show your work numerically, explain in sentences, or model using a different way.

Now, write each of the following division problems as equivalent multiplication problems. The first one has to be done to help you.

a) $20 \div 5$ b) $20 \div \frac{1}{5}$ c) $20 \div \frac{2}{5}$ d) $a \div \frac{b}{c}$ e) $\frac{b}{c} \div a$

$20 \bullet \frac{1}{5}$

What is the value of $\frac{2}{3} \bullet \frac{3}{2}$? What is the value of $4 \bullet \frac{1}{4}$?

These pairs of numbers are called *reciprocals*. What is the product of a number and its reciprocal?

Does every number have a reciprocal?

State the reciprocal of the following:

a) $\frac{5}{3}$ b) $-\frac{1}{2}$ c) 2000 d) $\frac{a}{b}$ e) 1.2 f) x

Write your own definition of a reciprocal below. Use the work you've done on this page to help craft the definition.

**Question 6: Negative Numbers**

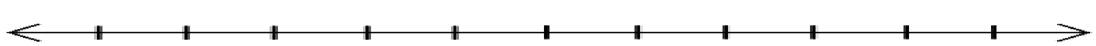
You are already familiar with operations involving positive numbers, but much mathematical work deals with negative numbers. Common uses include temperatures, money, and games. It is important to understand how these numbers behave in arithmetic calculations.

First, consider addition and subtraction. For each of the following, show how the answer can be visualized using a number-line diagram.

The air temperature at 2 pm was 12º. What was the air temperature at 8 pm, if it had dropped 15º by then?

Telescope Peak in the Panamint Mountain Range, which borders Death Valley, is 11045 feet above sea level. At its lowest point, Death Valley is 282 feet *below* sea level. What is the vertical distance from the bottom of Death Valley to the top of Telescope Peak?

In a recent game, I had a score of 3. I then proceeded to lose 5 points and 7 points on my next two turns. On the turn after that, however, I gained 8 points. What was my score at this moment in the game?



**Question 7: Negative Numbers and Temperature**

Consider the following question and subsequent work completed by a Golder student:

The temperature outside is dropping 3 degrees per hour. Given that the temperature at noon was 0º, what was the temperature at 1 pm? at 2 pm?

*At 1 pm, the temperature was -3° because 0+(-3) is -3. At 2 pm, the temperature was 0° because -3-(-3) is 0.*

Do you agree with the student's answer above? If yes, explain why. If no, explain why not.

What was the temperature at 3 pm? at 4 pm? at 7 pm?

Now, fill in the following table (the first two rows have been filled in for you):

| *Time* | *Hours since noon* | *Temperature change* | *Current temperature* |
|---|---|---|---|
| 12 | 0 | 0 | 0 |
| 1 | 1 | -3 | -3 |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |
| 6 | | | |
| *anytime* | *t* | | |

Now, using the information from the table above, try to create an equation that tells what the temperature is at *t* hours after noon. The table will help you if you consider the *number of hours since noon* and the *temperature change*. How do those two columns relate to the final column, *current temperature*?

**Question 8: Distributive Property, Continued**

You have seen that multiplication distributes over addition. Read the following work done by a Golder student who is answering if multiplication distributes over subtraction. Consider the student's work, and then use your own examples to answer the next questions.

*To answer if multiplication distributes over subtraction, I chose the following problem: 8(4 - 2). I am going to prove if*

$$8(4 - 2) = 8(4) + 8(-2)$$

*First, 8(4-2) can be written as 8(2) because 4-2=2. 8(2) is 16. The distributive property says that I can distribute the 8 to the 4 and then to the 2 and, in this case, subtract both products. If that also equals 16, then multiplication DOES distribute over subtraction.*
*So, 8(4) + 8(-2). 8(4) is 32 and 8(-2) is -16. 32 + (-16) is the same as 32-16, which equals 16. Therefore, multiplication does distribute over subtraction.*

Does multiplication distribute over multiplication?

Does multiplication distribute over division?

Now, create a *principle* (rule) for the distributive property. When can you use it? When does it work and when doesn't it work? Pretend you are teaching this to a classmate, so make your work clear and easy to understand.

Finally, solve the following problems using your work above. Show each step you take to solve.

a) 4(3 + 7) b) 5(9 ÷ 3) c) 7(5 – 4) d) 8(6 • 3)

**Question 9: Baseball, Evaluating Expressions and Conversions**

In baseball statistics, a player's slugging ratio is defined to be $\frac{s+2d+3t+4h}{b}$, where s is the number of singles, d the number of doubles, t the number of triples and h the number of home runs obtained in b times at bat.

Dana came to bat 75 times during the season, and hit 12 singles, 4 doubles, 2 triples, and 8 home runs. What is Dana's slugging ratio, rounded to three decimal places?

Many major-league baseball pitchers can throw the ball at 90 miles per hour. At that speed, how long does it take a pitch to travel from the pitcher's mound to home plate, a distance of 60 feet 6 inches? Give your answer to the nearest hundredth of a second. There are 5280 feet in a mile.

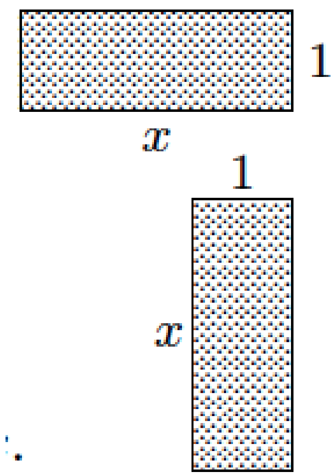
A Golder student was working on the question above and completed the following work:

*90 miles per hour = 90 miles per 60 minutes, which is 90÷60, which equals 1.5miles per minute. If the ball travels 1.5 miles in a minute and there are 5,280 feet in a mile, it travels 1.5x5280 feet, which is 7,920 feet in a minute. If there are 60.6 feet (60 feet 6 inches) between home plate and the pitcher's mound, then I would divide to find how long it takes to get to home plate. 7920÷60.6 equals 130.69 minutes.*

Do you agree with the student's work? Explain why or why not. Whether you agree or disagree, comment on what the student did well in his/her work after you explain your agreement or disagreement.

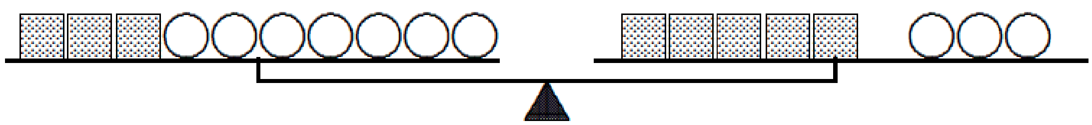
**Question 10: Grab Bag**

A rectangle whose length is x and whose width is 1 is called an x-block. The figure shows two of them.




**(a)** What is the area of an x-block?
**(b)** What is the combined area of two x-blocks?
**(c)** Show that there are two different ways to combine two x-blocks to form a rectangle whose area is 2x.
**(d)** Draw two different rectangular diagrams to show that x + 2x = 3x.

In the balance diagram below, find the number of marbles that balance one cube.



For each of the following, find the value of x that makes the equation true. The usual way of wording this instruction is solve for x :
**(a)** $2x = 12$ **(b)** $-3x = 12$ **(c)** $ax = b$

Percent practice: **(a)** 25% of 200 is what number? **(b)** 200 is 25% of what number? **(c)** Express 2/25 as a decimal; as a percent. **(d)** Express 24% as a decimal; as a fraction.

There are three feet in a yard. Find the number of feet in 5 yards. Find the number of yards in 12 feet. Find the number of feet in *y* yards. Find the number of yards in *f* feet.