Name: ____

Sophomore Summer Homework – Integrated Math 2 Honors/AP

Instructions:
This year's summer homework for math is meant to set us up for some great conversations in the first few weeks of school. This assignment will cover a variety of topics, including:

1) Skills you learned in Algebra 1 that you need to have down *really well*
2) New concepts that we will be talking about in the fall
3) Ideas that are just plain cool to think about!

You are expected to complete *all parts* of *every problem* to the best of your ability. Make sure to complete each section of each problem in order, showing all necessary work and explaining your answers in **complete sentences!** Read each problem *very carefully!*

<u>Time Requirement</u> – In order to be completed thoroughly, each problem should take between 30 minutes and 1 hour to finish. That means the entire assignment should take you **4 – 8 hours** to complete. Make sure to use the suggested "Summer Homework Calendar" for sophomores to you to help manage your time!

Why Should You Do This (and do it *well*)?

I know … you're *super excited* to do all of your summer homework! It's going to be the BEST part of your vacation! So you probably already know that this year's summer homework is extremely important for two reasons:

1) We will have a *quiz* over some of the concepts covered in this assignment sometime in the first week of class! You need to be ready!

2) These problems are going to be the **<u>foundation</u>** of AT LEAST the first two weeks of class. We will be using these problems to learn how to have whole-group discussions about mathematics.

*You **must** be prepared to participate, which means you <u>must</u> have your Summer HW on day 1! If you don't, you'll have to **sit alone & work silently** so that you can catch up with everyone!*

This is simply because in order to *participate* in the conversation, you must have something to **say!** You have to have put a good amount of *thought* into the problems—not just have rushed through them!

Name: ______________________________________

FAQs

What if I don't understand the problem? Re-read it and try to put it in your own words. Identify what you know, what you need to know, and what the problem is asking. If you're still stuck, leave it alone for a few days then come back to it. Still stuck? Go online and ask some questions in Google. *Still* stuck? Email your teacher. Coming to school in August with work incomplete and the excuse “I didn't know what to do” is unacceptable. It is your responsibility to seek out help if you need it. Your teachers are here, the Internet is helpful, you can talk to your classmates, your family, or a tutor.

What if I lost the homework? First, figure out how to get organized. Then, put your uniform on, come into school during office hours and ask for another copy. Finally, stay organized.

What if I don't want to do the homework? Too bad. This homework was carefully designed by Ms. Ceven, Mr. Crocker, & Ms. Burba and your work ethic is a reflection of your character. Everyone has to do things they don't necessarily want to do (does everyone want to get out of bed and go to work? Not always, but they do it anyway and then they get their paycheck. You do your homework and improve your brain functioning, get smarter, and get a better job later because you graduated from college). Now go do your homework.

What if I don't have a calculator? How do you not have a calculator? You needed one every day in Algebra this past year. Go get a calculator right now.

Resources

If you have trouble with a particular question, first make sure to read back through the rest of the work you've done in a problem. The problems are written such that the earlier parts should help with the later parts.

If you just can't figure something out, also feel free to e-mail Ms. Ceven (eceven@goldercollegeprep.org), Mr. Crocker (rcrocker@goldercollegeprep.org), or Ms. Burba (pburba@goldercollegeprep.org)!

Good luck! We'll see you in August!

Name: ____

Question 1: How do you like dem apples?

Sarah has 3 baskets, each containing 20 apples. Mike comes along and gives Sarah 10 more apples in total. To describe this scenario, a students writes $\mathbf{10 + 20 * 3 = 90}$, indicating that Sarah now has 90 apples.

Explain how the student came up with his or her equation & solution. Do you agree with the student's work? Why or why not?

Farmer Michelle has 6 baskets of 40 apples and 6 baskets of 20 oranges. To find the total number of fruits, Student #1 writes the expression $\mathbf{6(40) + 6(20)}$, while Student #2 instead writes $\mathbf{6(40 + 20)}$, and Student #3 writes $\mathbf{6 * 40 + 20}$.

Explain why Students #1 & #2 are both correct, even though they showed different work.

Explain why Student #3 is the only one who is *incorrect*.

Michelle needs to divide her total fruits among 3 grocery stores. To figure out how many fruits each store gets, she writes $\mathbf{6 * 60 \div 3}$.

In order for Michelle to correctly split up the fruits, should she multiply first or divide first in the expression above? Try both methods and compare the two.

You have previously learned about the Order of Operations (sometimes described using the acronyms PEMDAS and GEMS). Based on the work you did above, explain why it is so important for us to always perform different operations in a certain order.

Name: ____

Question 2: Walking the Line

The ***average*** of two numbers p and q is $\frac{1}{2}(p+q)$.

First, draw a number line and label it from zero to 10.

Choose two points on that line (mark them) and then evaluate $\frac{1}{2}(p+q)$ by substituting the values of those two points for p & q.

Using what you've just done, draw a new number line (do **not** label it!) & choose two new points to be p and q. Then, mark the new point where you would find $\frac{1}{2}(p+q)$.

Explain how you determined the location of the new point from the previous question.

Now, let's say you have three distinct points p, q, and r on a number line. Write an expression to represent the *average* of these values.

Draw a final number line (label this one from zero to 20), and label three points p, q, and r. Then, mark where you *think* the **average** would be located on this line.

Finally, calculate the average of your three values p, q, and r using the expression you wrote above and mark this point on the number line you just drew. How does this point compare to where your guess is located

Be prepared to discuss how the average of p, q, and r compares to the average of p and q.

Name: ____

Question 3: Equivalence & *U*

$$\frac{2u-6}{u-3}$$

Evaluate the expression above when $u = 5$, $u = -3$, and $u = \frac{1}{8}$. You should notice a pattern in your answers—if not, go back and check your work! (#Uwrong)

Now simplify the expression above to make this pattern even easier to notice! Hint: It involves *factoring* the numerator!

Explain how your simplified expression relates to the answers you got in the first part of this problem.

What is (almost) always true about the original expression $\frac{2u-6}{u-3}$?

There is actually one value for *u* that is an exception to the answer to the previous question. Determine that value and <u>explain why it is an exception</u>.

Name: ____

Question 4: Going the Distance

Instead of walking along two sides of a rectangular field, Janely took a shortcut along the diagonal, thus saving distance equal to half the length of the longer side. Find the length of the long side of the field given that the length of the short side is 156 meters.

Draw a picture to represent this situation. Make sure to label all of the known distances and unknown distances.

Write an expression to represent the length of the distance she would walk along the sides of the rectangle, instead of cutting across the diagonal.

Write an expression to represent the distance she would *save* by walking along the diagonal? (Remember that she saves "half the length of the longer side" in this case!)

Now write an expression to represent the length of the distance she would walk along the diagonal (in terms of the length of the longer side).

Finally, use what you have done to solve the problem!

Name: ____

Question 5: Counting to a Billion

How long would it take you to count to one billion, reciting the numbers out loud one after another? Write a quick guess.

Explain your guess from above. What information did you use to decide how long it would take to count to one billion?

Now, let's use math and logic to come up with a thoughtful answer. First, think about how long it takes to say **each** number individually. Does it take the same amount of time to say each number? Explain why or why not.

Now, consider how many numbers there are between one and one billion. Using the time or times you decided it takes to say each number, determine how long it should take to say *all* the numbers in order.

Finally, take a look at the units you used in your answer. Is your solution in seconds, minutes, hours, or something else? Would your answer make sense to other people? If not, convert your number into a more manageable answer that would make more sense to others. (Example: If I tell you I'll see you in 86,400 seconds, then you probably wouldn't immediately understand what I mean. If I tell you I'll see you *tomorrow*, which is 86,400 seconds away from now, then you'd get it right away.)

Name: ____

Question 6: Time Flies…

Consider the clock shown to the right. The minute hand completes one rotation around the clock every hour.

Suppose that you see a small bug named Marvin sitting on the very tip of the minute hand and another bug named Sid sitting a few inches away from him. You leave the room, but *twelve hours later* you see that Sid & Marvin stayed on the minute hand as it moved.

Which bug has traveled the most *distance* in this time? Which bug has traveled the most *rotations*? Explain.

Suppose you look at the clock at 5:00 PM and see that Marvin is still on the minute hand's tip; however, Sid has jumped to the hour hand. How many *rotations* has each bug completed at…

8:00 PM? **2:30 AM?** **4:45 AM?**

Instead of talking about "numbers of rotations," we generally measure rotations in **degrees**, where *one full rotation* is equal to *360 degrees* or 360^o. (So, for example, rotating in three complete circles is the same as traveling 1080^o, because $3 * 360^o = 1080^o$.) Convert each of your answers to the previous problem into *degrees* and write your new answers below.

Finally, suppose that Sid's British cousin Bert is at Big Ben, an enormous clock tower in London. Bert is sitting on the tip of Big Ben's minute hand, which is 14 feet long. Sid is still on the minute hand of his small, normal clock. Both Sid & Bert stay on their clocks from 3 PM to 6 PM.

During this time, who travels the most distance? Who travels the most rotations? Explain.

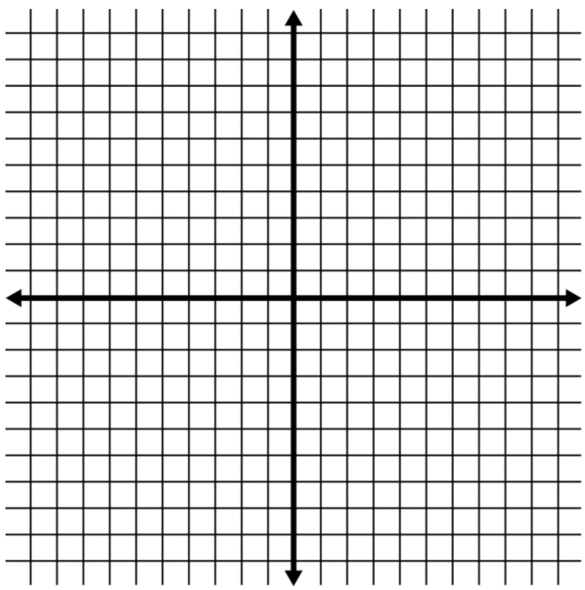
Name: ______________________________________

Question 7: Doppelgängers

A **solution** to a linear equation is a point (x, y) that makes the equation *true* when you plug in the values of x and y. For example, the point $(-2, 3)$ is a solution of the equation $6x - 3y = -21$, because substituting -2 for x and 3 for y gives you $-21 = -21$. On the other hand, the point $(5, -1)$ is *not* a solution to the equation, because substituting 5 for x and -1 for y gives you $33 = -21$.

Consider the equations $2x + 3y = 6$ and $4x + 6y = 12$.

Graph both lines on the coordinate plane shown below.

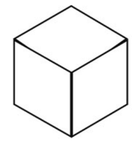


Now, substitute the coordinates $(0, 2)$ into both equations. Then, substitute the coordinates $\left(-4, \frac{14}{3}\right)$ into both equations. What do you find? Explain how this relates to the graph you completed above.

Remember that a system of linear equations involves two or more linear equations. Such systems can have *one solution, no solution, or infinite solutions.* Explain which of these cases applies to the system above.

Name: ______________________________________

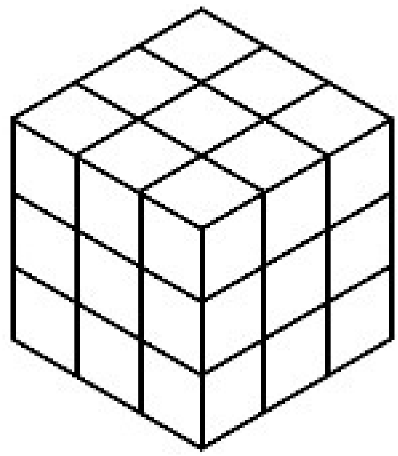
Question 8: Cubism (don't ask Picasso)



Suppose you have a cube (to the right) with side lengths measuring 1 cm.

What is its *surface area*? What is its *volume*? (Make sure to include correct units!)

Now suppose you stack together cubes identical to the one above so that it forms a larger cube as shown below.

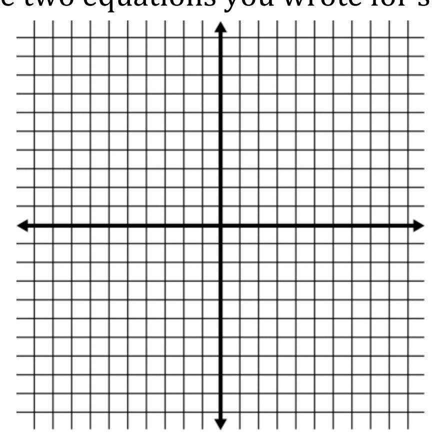


What is the surface area of this cube? What is its volume?

Now, suppose that at one corner of the larger cube we cut away one of the smaller cubes. What is the surface area of the remaining solid? What is its volume?

When is the numerical value of the surface area of a cube equivalent to the numerical value of its volume? Is there ever a time when this occurs? (Hint: Consider a cube with side lengths measure *x* cm. Try to write an equation that compares the cube's surface area & volume and solve for x. You may need to factor!)

Extension: Try to graph the two equations you wrote for surface area & volume above. What do you notice?



Name: ____

Honors/AP Students Only

These questions are modified AP Calculus questions. They are straight from an AP exam given to high school juniors/seniors within the past 7 years.

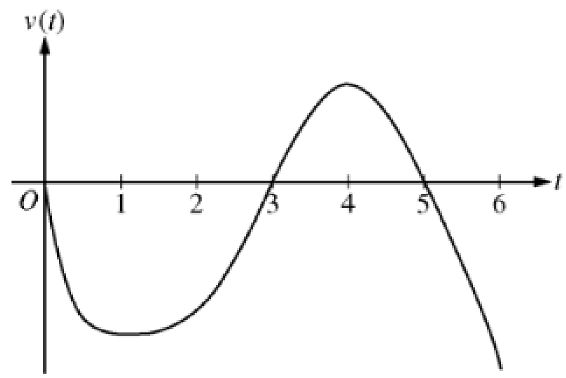
Honors – choose 1 of the 2 problems
AP – complete both problems
Show the work you need to show so that others understand where you got your answer, take your time, and have fun.

Question 9: Particle Schmarticle

PRACTICE FREE RESPONSE QUESTION

AP CALCULUS 2008 #4

Question 4



Graph of v

A particle moves along the x-axis so that its velocity at time t, for $0 \le t \le 6$, is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 0$, $t = 3$, and $t = 5$, and the graph has horizontal tangents at $t = 1$ and $t = 4$. The areas of the regions bounded by the t-axis and the graph of v on the intervals $[0, 3]$, $[3, 5]$, and $[5, 6]$ are 8, 3, and 2, respectively. At time $t = 0$, the particle is at $x = -2$.

(a) For $0 \le t \le 6$, find both the time and the position of the particle when the particle is farthest to the left. Justify your answer.

(b) For how many values of t, where $0 \le t \le 6$, is the particle at $x = -8$? Explain your reasoning.

(c) On the interval $2 < t < 3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

(d) During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.

Hints:

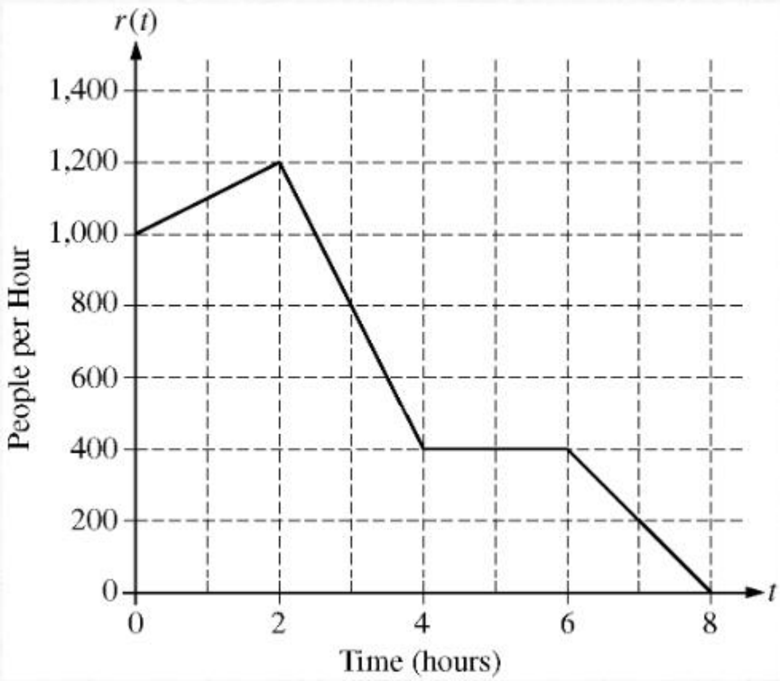
- Area under curve above x-axis is positive distance travelled. Area under curve below x-axis is negative distance travelled (other direction—negative usually means "to the left").
- Speed is the absolute value of velocity. (Velocity can be positive or negative depending on direction. Speed is just a number without direction.)
- Acceleration is represented by the slope of velocity.

Name: ______________________________

Question 10: A Day at the Amusement Park

PRACTICE FREE RESPONSE QUESTION

AP CALCULUS 2010 #3 (Modified)



There are 700 people in line for a popular amusement-park ride when the ride begins operation in the morning. Once it begins operation, the ride accepts passengers until the park closes 8 hours later. While there is a line, people move onto the ride at a rate of 800 people per hour. The graph above shows the rate, $r(t)$, at which people arrive at the ride throughout the day. Time t is measured in hours from the time the ride begins operation.

(a) Assuming area between the function and the x-axis represents total number of people, determine how many people arrive at the ride between $t = 0$ and $t = 3$ hours? Show computations that lead to your answer.

(b) Is the number of people waiting in line increasing or decreasing between $t = 2$ and $t = 3$ hours? Justify your answer. (THINK about this before jumping to conclusions.)

(c) At approximately what time t is the line the longest? About how many people are in line at that time? Justify your answers.

(d) Estimate a time t at which there is no longer a line for the ride. Explain your reasoning.